# B.M.S COLLEGE FOR WOMEN AUTONOMOUS <br> BENGALURU - 560004 

## END SEMESTER EXAMINATION - OCTOBER 2022

## M.Sc. in Mathematics - II Semester <br> Basic Statistical Methods

## Course Code: MM206S

Duration: 3 Hours

QP Code: 21006
Max marks: 70

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. a) A random variable $X$ has following probability function:

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p(x)$ | 0 | $k$ | $2 k$ | $2 k$ | $3 k$ | $k^{2}$ | $2 k^{2}$ | $7 k^{2}+k$ |

(i) Find $k$,
(ii) Evaluate $P(X<6), P(X \geq 6)$ and $P(0<X<5)$,
(iii) Let $P(X \leq a)>\frac{1}{2}$. Find the minimum value of $a$.
(iv) Determine the distribution function of $X$.
b) A two-dimensional random variable $(X, Y)$ have the joint pdf $f(x, y)=8 x y$, where $0<x<y<1$.
(i) Find $P\left(X<\frac{1}{2} \cap Y<\frac{1}{4}\right)$.
(ii) Find the marginal and conditional distribution.
(iii) Are $X$ and $Y$ are independent?
2. a) Prove for any two random variables $X$ and $Y$
(i) $E(+Y)=E(X)+E(Y)$ and
(ii) $E()=E(X) E(Y)$ where $X$ and $Y$ are independent random variables.
b) Find mean and variance of normal distribution.
3. a) Fit a Parabola $y=a x^{\wedge} 2+b x+c$ to the given data

| $x$ | 1 | 3 | 4 | 6 | 8 | 9 | 11 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 1 | 2 | 4 | 4 | 5 | 7 | 8 | 9 |

b) If F is the force required to lift a load W , by means of a pulley, fit a linear expression $F=a+b W$ against the following data:

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QUESTION PAPER

| $W$ | 50 | 70 | 100 | 120 |
| :---: | :---: | :---: | :---: | :---: |
| $F$ | 12 | 15 | 21 | 25 |

4. a) Calculate the coefficient of correlation from the following data; given ranks of 10 students in English and Mathematics.

| Rank in English | 3 | 1 | 5 | 4 | 2 | 6 | 8 | 10 | 9 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Rank in Mathematics | 2 | 4 | 3 | 1 | 5 | 10 | 7 | 9 | 8 | 6 |

b) Find the M.g.f of binomial distribution.
5. a) There are two bags. First bag contains 5 red, 6 white balls and the second bag contains 3 red, 4 white balls. One bag is selected at random and a ball is drawn from it. What is the probability that it is (i) red (ii) white.
b) State and prove Boole's inequality.
6. a) State and prove Baye's theorem.
b) Define Bernoulli distribution and find its mean, variance and mgf.
7. a) A random sample of size 16 has 53 as mean. The sum of squares of deviations from mean is 135 . Can this sample be regarded as taken from the population having 56 as mean? Also obtain $95 \%$ confidence limits for the mean. (at $5 \%$ level $t$ value for 4 degree of freedom $=2.13$ ).
b) Samples of sizes 10 and 14 were taken from two normal populations with S.D 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at $5 \%$ level. (at $5 \%$ level for 22 degree of freedom $t_{0.05}=2.0739$ )
8. a) The theory predicts the proportion of beans in four groups $A, B, C, D$ should be in the ration 9:3:3:1. In experiment with 1600 beans the numbers in the four groups were $882,313,287$ and 118. Does the experimental result support the theory? $\left(\chi^{2}\right.$ value at $5 \%$ level of significance for 3 degrees of freedom $=7.815$ ).
b) In a sample of 8 observations, the sum of squared deviations of items from the mean was 84.4 . In another sample of 10 observations, the value was found to be 102.6 . Test whether the difference in variance is significant at $5 \%$ level.
(You are given that at $5 \%$ level critical value of $\mathrm{F}(7,9)$ degree of freedom is 3.2927 ).

